Measuring the qualities of Choisy's oblique and axonometric projections

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Auguste Choisy is renowned for his «axonometric» representations, particularly those illustrating his *Histoire de l'architecture* (1899). Yet, «axonometric» is a misnomer if uniformly applied to describe Choisy's pictorial parallel projections. The nomenclature of parallel projection is often ambiguous and confusing. Yet, the actual history of parallel projection reveals a drawing system delineated by oblique and axonometric projections which relate to inherent spatial differences. By clarifying the intrinsic demarcations between these two forms of parallel projection, one can discern that Choisy not only used the two spatial classes of pictorial parallel projection, the oblique and the orthographic axonometric, but in fact manipulated their inherent differences to communicate his theory of architecture.

Parallel projection is a form of pictorial representation in which the projectors are parallel. Unlike perspective projection, in which the projectors meet at a fixed point in space, parallel projectors are said to meet at infinity. Oblique and axonometric projections are differentiated by the directions of their parallel projectors. Oblique projection is delineated by projectors oblique to the plane of projection, whereas the orthographic axonometric projection is defined by projectors perpendicular to the plane of projection. Axonometric projection is differentiated relative to its angles of rotation to the picture plane. When all three axes are rotated so that each is equally inclined to the plane of projection, the axonometric projection is isometric; all three axes are foreshortened and scaled equally. Dimetric projections have axes rotated so that two are equally foreshortened and thus scaled by the same factor, and trimetric projections have three differently inclined axes. One trait that overtly divides oblique from axonometric projection is

that oblique projection has both two-dimensional and three-dimensional space attributes. Oblique projection presents an aspect of two-dimensional geometric purity stemming from the fact that one plane of the object lies parallel to the picture plane. Depth is indicated in an act of oblique extrusion—from the plan in the case of military projections and from the elevation, or section, in cavalier projections. The visual cue is that the orthogonal geometry of a plan, elevation, or section is un-distorted, as it lies parallel to the picture plane. Choisy manipulates these spatial differences within his *Histoire de l'architecture*. Whereas the abundant oblique projections systematically build a measured space-form language through their orthographic plane, the few true axonometrics in the *Histoire* model the spatial effects of singular architectonic assemblages.

Parallel Projections: the oblique and the orthographic axonometric

One can locate Choisy's academic introduction to parallel projection to a course on Descriptive Geometry taught by Jules de la Gournerie at the Ecole Polytechnique. Indeed, the instruction can be pinpointed to the day. Dated Monday, November 9, 1861, Gournerie's summary of lesson 15 reads: «Perspective axonométrique. Notions générales, perspective d'une niche». Lesson 16, on Friday, November 13, followed with «Perspective isométrique-Rapporteur isométrique-ellipses isométrique. Perspective cavalière-principe-application: cylindre, sphère, niche» (De la Gournerie 1861–3). This content follows that found in Gournerie's *Traité de Géométrie Descriptive* (1860–4). In the first part of this treatise, published in 1860, the brief, but entire, fourth book is devoted to «Perspectives Axonométrique et Cavalière». Despite the dated, ambiguous terminology, Gournerie's organization of parallel projection is a twofold division between the orthographic axonometric and the oblique.

In the forward to this *Traité*, Gournerie identifies the dual influences of the English and the Germans in the development of axonometric «perspective». He singles out Englishman William Farish as the catalyst with his «Perspective isométrique» published in 1822 and credits two German sources as instrumental to his own refinements: Meyer and Meyer's *Lehrbuch der Axonometrie* (1852, 1853, 1855, 1863) and Robert Schmidt's *Theoretisch-praktischer Lehrgang der Axonometrie* (1859). Gournerie's organization and guidance of parallel projection closely follows the principles propounded by the Germans Meyer and Meyer.

The evolution of axonometry

William Farish published his ground-breaking drawing method «On Isometrical Perspective» in 1822. Isometric literally means equal measure and Farish indi-

cates that isometrical perspective describes a cube represented so that the three principal faces make equal angles with the picture plane and so consequently all have at the same scale (Farish 1822, 4). Farish acknowledges that the isometric perspective is not of course a perspective, but corresponds to the function of a perspective by rendering a pictorial view. The eye of the artist is placed at an indefinite distance, rather than at a fixed point. Regardless of his expressive terminology, Farish understood isometrical perspective as a «species of orthographic projection;» the projection is on a plane perpendicular to the diagonal of a cube (Farish 1822, 5–6).

Soon after Farish first published his account, the method was quickly engaged by German engineers, geometers, and mathematicians. In 1844 the critical geometric, trigonometric evolution of isometry toward axonometry was achieved and published by Julius Weisbach in a brief scientific article, «Die monodimetrische und anisometrische Projectionsmethode (Perspective)». In addition to building on the many isometrical methods successive to Farish, Weisbach expanded on the nomenclature and theories of crystallography proposed by Carl Friedrich Naumann in 1830 to establish a projective axis system. Weisbach theorized that points could be projected by their coordinates on these axial planes. Thus, the orientation and reduction of the axes did not have to stay isometrically inclined and equally reduced. These coordinate axes could be reduced in scale relative to their angles of inclination and these conditions qualified as monodimetric, the case in which two axes are equally reduced or anisometric, in which all three axes are differently reduced. While Weisbach differently coordinated the space of the isometric, he did not use the term axonometric to encompass these newly defined axial projections. This appears to have occurred eight years later, in 1852, in a work by the Meyer brothers of Freiburg.

M. H. Meyer and C. Th. Meyer's serialized work *Lehrbuch der Axonometrie oder der Gesammten isometrischen, monodimetrischen, und anisometrischen Projectionslehre* appeared in 1852, 1853, and 1855. A fourth section was anticipated for 1856; however, it was finally published and bound with the first three sections in 1863 as *Lehrbuch der axonometrischen Projectionslehre*.[1]

The foreword and introduction to Meyer und Meyer's *Lehrbuch der axonometrischen Projectionslehre*, attributable to 1852, give a brief history of parallel projections and leads up to their establishment of the term, axonometric. They categorize the basic geometric delineations and methods, describing the English and German scholars responsible for the evolving lineage of the isometric methods. They single out their compatriot Professor Julius Weisbach for his innovative work in which axis rotation allows for more natural relations between the form and its representation in parallel projection. Meyer and Meyer propose that this axis system, comprised of isometric, monodimetric, and anisometric

projections, be named axonometry, and that the method be called axonometric projection (Meyer and Meyer 1855–63, 8). Whereas Weisbach's work was exclusively theoretical, Meyer and Meyer comprehensively explore axonometric projection's theory and practice through its history, aesthetics, geometry, mathematics, process, and application. Their explication is extensively illustrated with both in-text demonstrations and appended graphic illustrations.

The Meyers aimed to make abstract geometry practical (Meyer 1855–63, 12). They do not posit a single method by which to structure axonometric representations. They reason that one can find the axis-system both by calculation and by drawing (Meyer 1855–63, 87). Chapters are dedicated to prove and demonstrate the axis-system first mathematically via trigonometry, but also geometrically via axial coordinate projections. The Meyers axonometric methods are clearly in the realm of analytic geometry, as first posited by Weisbach, and not in that of Mongeon descriptive geometry (Loria 1921, 414). They submit that such descriptive geometrical methods require excessive steps by generating the projected representation from two planes, and so first require geometrical drawings of both the plan and the elevation (Meyer 1855–63, ix). The Meyers construct their axonometric projections in space; the representation is bound to axiality. Axonometric drawing methods are demonstrated by first determining the desired orientation of the axonometric axes, and by consequence the orientation of the inclined object in space. Foreshortening ratios are shown to be adduced trigonometrically, geometrically, or estimated via a tabulated system for those more inclined to application than reason (Meyer 1855–63, 18).

The brothers submit that the axonometric method is not only a means by which to manifest an image of real bodies in a single picture, but that the picture has the same effect as the object itself (Meyer 1855–63, v). Unlike the inherent fragmentation of multi-planar orthographic projections, the axonometric assembles. It is, «a representation that permits simultaneously a deeper view into the mechanism, its relative relationships and true magnitude» (Meyer 1855–63, vorwort).[2] Optical perception does not adequately allow for such analytic perception. Meyer and Meyer challenge that the axonometric is truly spatial. Points, lines, surfaces, and bodies as located in space are projected on one picture plane (Meyer 1855–63, 1). The values of true form and size are juxtaposed in space. Under axonometric projection, the figurative representation on the plane of projection allows one to understand the true spatial nature of bodies. They juxtapose this spatial view of parallel projection to that of conic perspective, stating: «According to whether one gives the eye a definite standpoint, or one applies certain appropriate rules, one differentiates between perspectival projections and free or imaginative projections» (Meyer 1855–63, 1).[3] The Meyers argued that central perspective, or the «painter's perspective», finitely restricts the image to the eye.[4]

On the other hand, the axonometric projection, by virtue of its true parallelism in infinite space, «allows the eye to hover at each single point» and is thus liberated and imaginative (Meyer 1855–63, 1).[5] The invented views fabricated by axonometric projection are contingent on a different conception of the object and man *in space*.

Like Naumann and Weisbach before them, the Meyers advocated that the nature of the object should influence the orientation of the axis system to the picture plane and thus its representation. To establish the orientation of the axis-system the object is imagined as to how it naturally falls into the picture, or projection, plane. They clearly assert that maintaining this natural aspect will more closely approximate the image as one receives it to the eye, the first prerequisite for image making (Meyer 1855–63, v). The vertical axis coordinate plane typically stays vertical as it is equated to gravity; it measures height. The perpendicular, horizontal planes thus naturally correspond to the plan; these coordinate planes parallel its length or width (Meyer 1855–63, vii). They observe that the easily constructed and accurate isometric system can be an awkward representation for some objects due to its prescriptive, inclined, often bird's-eye, view (Meyer 1855–63, vi). The natural relation of the object to the eye is not considered in the isometric due to its completely systematic application. The Meyers assert that a more complete expression is offered by the mondodimetric and anisometric axis-systems which support a closer interaction between observer and object through the rotation of their axes.

The Meyers construct their argument for the axonometric visually as well as verbally. Their first two plates juxtapose isometric, monodimetric, and trimetric cubes with both their ratio of reduction and their axis system. There are nineteen incremental demonstrations of monodimetric rotations and thirteen anisometric rotations (Meyer 1855–63, plates 1, 2: figs. 2–32). The visual succession of cubes systematically expresses the integration of mathematical ratio, axonometric axis, and phenomenal expression. The representational choice between iso-, monodi- or aniso-metric can reflect, heighten, or diminish the perception of particular aspects of any given object.

Establishing a point of view, however, does not conclude with the selection of an axis-system. Within the same axis system and ratio of reduction, there can be varied points of view. The inherent multi-dimensional spatiality of an axis system allows reversibility as demonstrated in Figure 1 (figs. 36–37). Meyer and Meyer demonstrate this with a given monodimetric axis-system with a ratio of 1/2:1:1. The brothers observe, «One can also exert an inverse bias with this axis-system, so that the observer may see into the system as at an angle or from below» (Meyer 1855–63, 50).[6] Meyer Figure 36 of the illustration highlights the potential for transpositions within the same axis system and the same ratio via the axonomet-

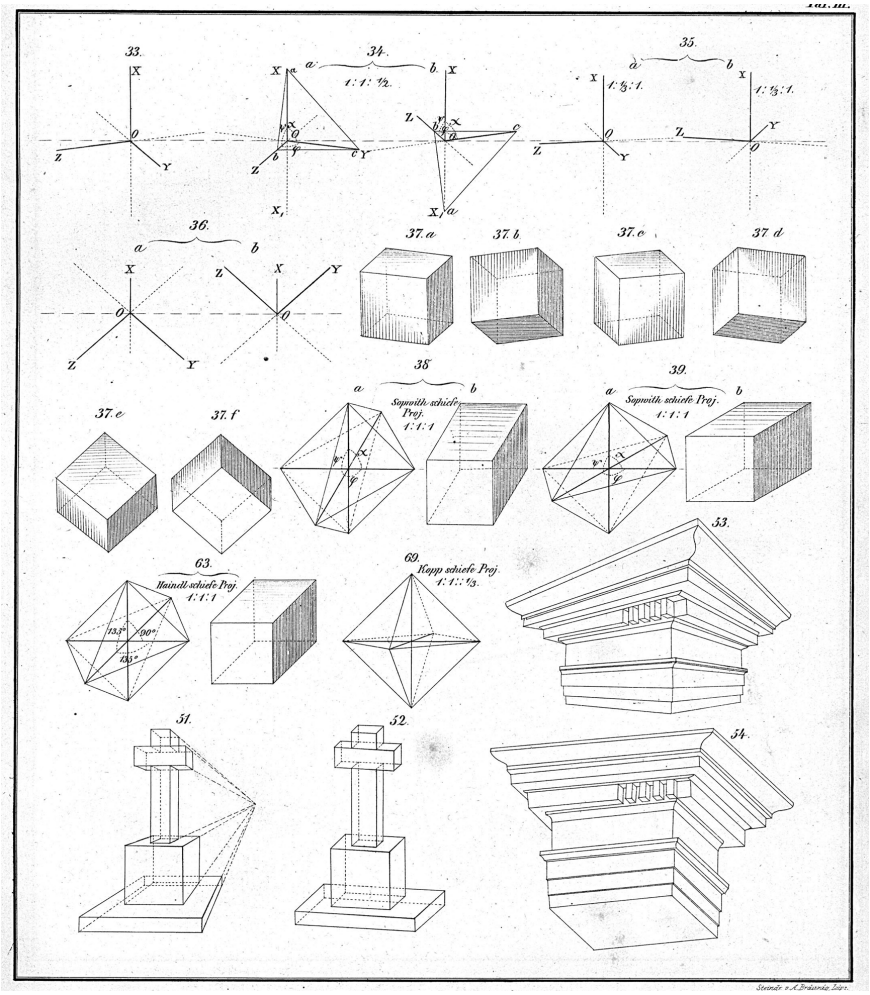

Figure 1
(Meyer and Meyer [1863] 1852, 1853, 1855, pl. 3)

ric lines; first they are articulated forward, then backward. Meyer Figure 37 shows views from six «different positions» of a cube within this same monodimetric axis system. The cube is seen as below, above, and to the side. The related views are reflected in relational changes within the same ratio of reduction.[7] The rotational freedom of the axonometric allows the drawing's creator to place the viewpoint of the observer in the most suitable position.

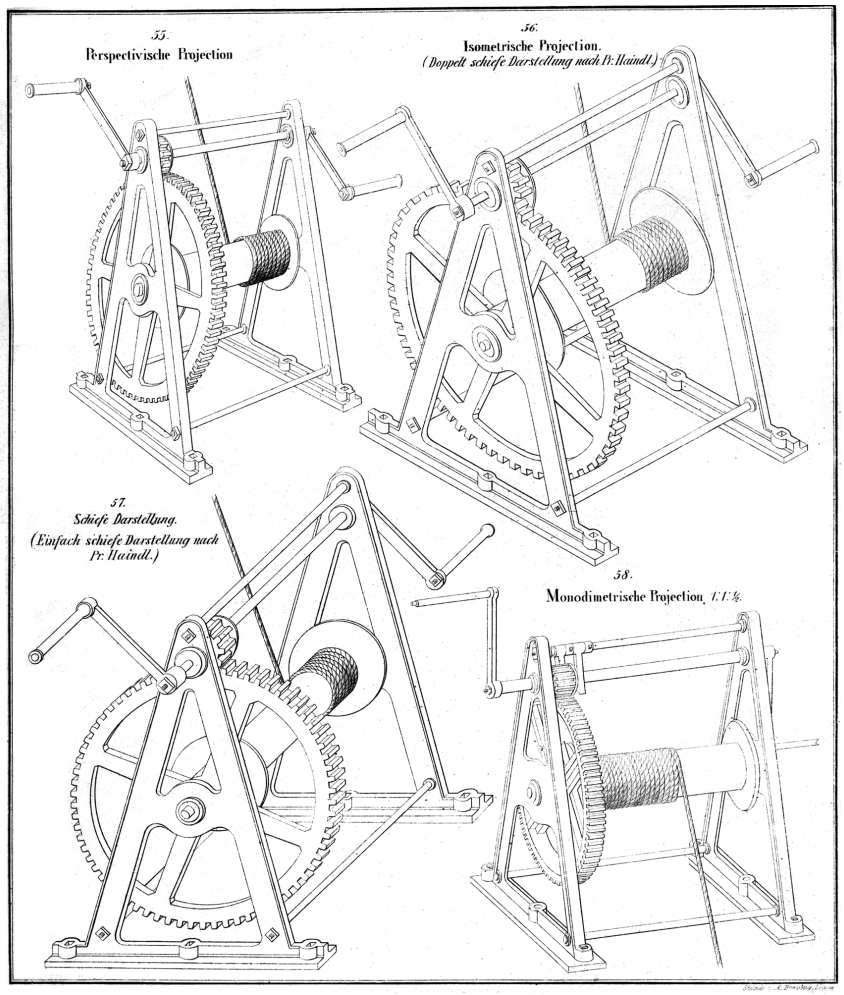

Figure 2
(Meyer and Meyer [1863] 1852, 1853, 1855, pl. 7)

The use of comparative illustrations broadens to demonstrate this point. Figure 2 (Meyer 1855–63, plate 7) contains four representations of a reel, «a perspective projection, an isometric projection, an oblique representation, and a monodimetric projection».[8] The inclinations of the perspective and monodimetric drawings to the picture planes are closely approximated in order to better make comparisons (Meyer 1855–63, 58). The Meyers argue that the monodimetric rep-

resentation so closely approximates the perspective that the impression is the same. Indeed, they avow, distortion brought about by the convergence of the perspective's projectors introduce some doubt as to the reality of the machine. Furthermore, they assert that the practical aspects of the monodimetric should be obvious (Meyer 1855–63, 57). It is within the context of perspective and monodimetric comparison that we see their only worm's-eye view representation, Meyer Figure 54 in Figure 2. The reverse axis, using the same monodimetric relation as that conveyed in the demonstration cube, is used to convey a matching image to the up-view perspective of an Ionic entablature. Again, the Meyers conclude that the rendered impressions were similar but the tectonic up-view is more consequential than the eye view (Meyer 1855–63, 57–8).

The brothers differentiate as well the axonometric views from oblique projection, often referenced as cavalier perspectives (Meyer 1855–63, 8–9). In the axonometric projections the reel is tilted via coordinate planes oblique to the plane of projection and the projection lines are perpendicular to the projection plane. In the oblique projection the body is parallel to the projection plane and the projection lines are 45 degrees oblique (Meyer 1855–63, 9).[9] They comment, «the picture developed by the oblique projection is like an orthographic shadow» (Meyer 1855–63, 2).[10] The resultant images from the two systems are different and one sees different things. Meyer and Meyer concede that oblique projections are easier to construct than axonometric projections since the projection plane is parallel to, or coinciding with, one of the coordinate planes (Meyer 1855–63, 59–60). Within this plane, and parallel to it, there is simple geometric truth; circles are circles and right angles are right angles. Yet, while the axonometric method allows one to explicitly determine the foreshortening scale of any chosen inclination given to the oblique projectors from this orthogonal plane, the angle itself is nevertheless always arbitrarily or conventionally selected (Meyer 1855–63, 60). It is thus not relationally exact, as in the space of axonometry. They resolve that the inherent geometrical orthogonality of oblique representations relate more like the diagram, than the form.

Being the first to posit the theory, Weisbach could not have been expected to envision its practical utility. However his work in 1857, *Anleitung zum axonometrischen Zeichnen,* is clearly a response to the theory and practice of axonometry as a method of representation as advanced by the Meyers. Weisbach appends only two plates and an apology that the practical explication was not as complete as one might wish (Weisbach 1857, v). Yet, within Weisbach's book there is a note of originality; seven figural geometrical constructions demonstrate the projection of axonometric shadows as evidenced in Figure 3. Gino Lorio asserts that Weisbach's application of shadows firmly establishes the axonometric's place as a method of representation and not merely a mathematical or geometrical operation (Loria 1921, 419).

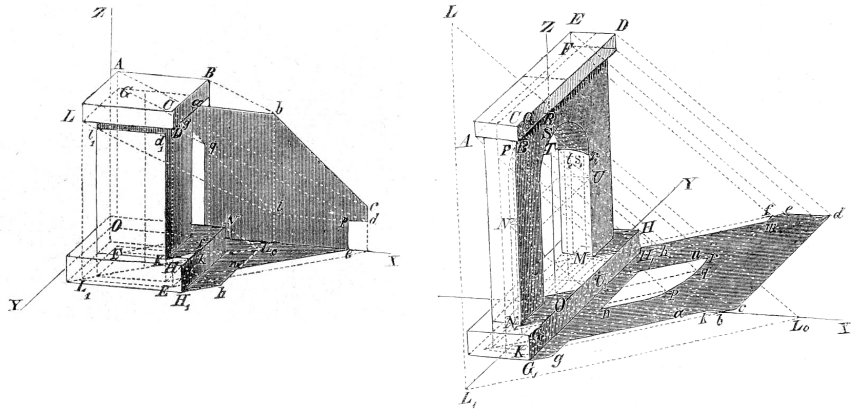

Figure 3
Two in-text demonstrations from Weisbach 1857

The extraction of axonometry in the mid eighteenth century further delineates parallel projection. An inherent difference between axonometric and oblique projections is made apparent: one manifests a true spatial representation in and of three-dimensional space and the other offers a synthetic picture via shadow, diagram, and two-dimensional extrusion. The Meyers allow that the axonometric projection is a spatially imaginative view. The implicit bird's-eye view is questioned. The Germans fundamentally shift the space of representation. The axonometric method now allows for observer specific orientations, reversible views, and analytic techniques, all of which can be variably used to impart a subjective point of view to the accurate spatial representation of the tectonic object. The domain of parallel projection is paradigmatically changed.

The projected structure of vaults

While Choisy's technical introduction to parallel projection can be directly traced to Gournerie, and by extension the Meyer brothers,[11] credit for Choisy's technique can be attributed to Cambridge mechanical philosopher and architectural historian, Robert Willis.

When Choisy was awarded the Royal Gold Medal from the Royal Institute of British Architects in 1904, he prominently acknowledged the influence of the Reverend Willis' work. Choisy recounted an early and memorable encounter with a Willis essay, «On the Construction of Vaults in the Middle Ages» (Choisy 1904,

452). The article, first published in the *Transactions of the Royal Institute of British Architects* in 1842, was very quickly translated into French by César Daly and published in the *Revue Générale de l'architecture et des travaux publics* in 1843.[12] The essay is a critical examination of the English vaults of the middle ages in which Willis expertly and minutely elucidates their tectonic relationship of geometry, construction, and form; Willis' discourse expands as a progressive exegesis of form which is paralleled by three plates of isometric drawings, figure 4. Within Willis' essay the isometric made its public debut as a privileged view for comparatively exhibiting architectural structure.[13]

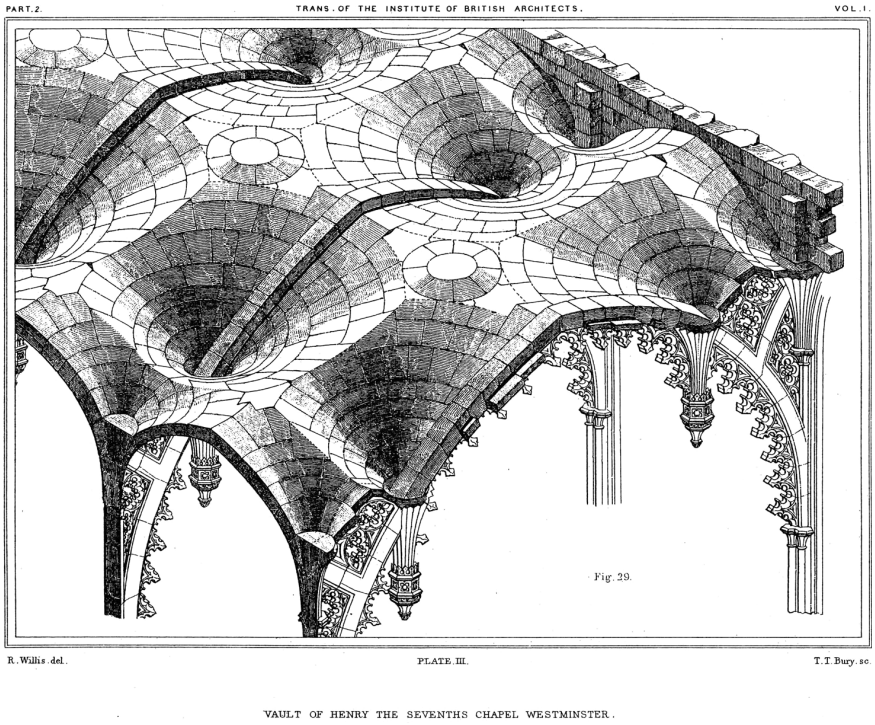

Figure 4
Westminster, fan vault (Willis 1842, pl. 3)

A close examination reveals an undeniable parallel in subject matter, analytic method, and particularly graphic communication between Willis' essay and Choisy's early publications on Roman and Byzantine vaults. Willis rationally yet

emotively expressed the mechanical essences of particular exemplars of fan vault construction using shaded isometric renderings. The graphic constructs manifest the mechanical yet corporeal relationship between stones, as component parts, and their assembly as vaulted structures similar to those seen in Choisy's first book, *L'art de bâtir chez les Romains* (1873).

While Choisy was unconditionally inspired by Willis' methods, there are two notable distinctions between Willis' and Choisy's use of parallel projection: projector orientation and point of view. Willis' drawings are almost exclusively isometric drawings from a bird's-eye view, while Choisy's representations vary between oblique and axonometric projections from a worm's-eye view.

Revolutions in space

The bird's-eye view had been an implicit convention of both the oblique and isometric projections. The German's alternatively referred to the cavalier (from the face) oblique projection as a *Vogelperspektive*, a bird's perspective. Military oblique projections were conceptually tied to an over-view of the horizontal plane through the orthogonal plan and its volumetric extrusion. Farish and Sopwith's isometrical convention also assumed the perspective normatively from a bird's-eye view. Jopling maintained a bird's-eye view as he rotated geometric solids within his static isometric cube-frame. The underside is depicted by rotating the object, not the bird's-eye position of the observer. Furthermore, all of Willis' isometric representations are from above. The bird's-eye view was assumed. Weisbach and the Meyers both commented on the alien up-standing view accorded to isometrical projection. The convention of parallel projections, oblique or isometric, held the observer at a distance, defining the object by a summary over-view. The introduction of a truly spatial representation shattered the extant conventions, opening up new ways to see. The geometric shift from the isometric cube to the axes of coordinates stimulated re-interpretations of all parallel projections. The spatio-geometric development of axonometry allowed not only for Choisy's re-conception of Willis' isometry, but of oblique projections as well.

The geometry of axonometric systems, as ultimately resolved by the Germans in the mid 1850s, clearly asserted free rotation and precise three-dimensional extension in the pictorial construction of the tectonic object. Consequently, the essential question in establishing a representation in this new axial space was the notion of point of view. The Meyers, in particular, introduced the notion of aesthetic parity between perspectival and axonometric representation. Axonometric space allowed for the tectonic truth to be materialized relative to a positioned observer.

Choisy had a clear intention to express the tectonic nature of Roman vaulted construction as Willis had done with the English fan vaults, however Choisy

could not tectonically explicit the effect of Roman vaults from above. The primary material structure of Roman vaulted construction consisted of an «encased framework in a mass of masonry» (Choisy 1873, 3).[14] Tectonically speaking, the lower, inside surface is where the embedded framework, this essential component, is structurally expressed. Thus, the necessitating nature of the Roman tectonic combined with the spatio-geometric potential of axonometry led Choisy to revolutionize the tectonic vision of parallel projection via a view from below.

The up-view dimetric portrayed by Meyer and Meyer establishes a view of an Ionic entablature as by an observer looking up, analogous to the perspectival view to which it is juxtaposed (Figure 1, Meyer 1855–63, figs. 53–54). While their perspective view's bottom boundary is delimited by its leading edges meeting at the corner, the dimetric view reveals an underside. However, the Meyers' representation inconsequentially portrays the underworld. This is largely due to an axial orientation which favors the front elevation; the drawing is very close to an oblique perspective from the front elevation.[15] Their chosen axial orientation flattens the representation and depth is alluded to by an oblique angle approximating a conventional cavalier drawing.[16] This axonometric drawing, so close to a cavalier, is decidedly unspatial. However, the Cartesian concept of space would eventually be used to focus attention on the spaces of structure and structures in space. Choisy's up-views manipulate the rotational space of the axonometric to move the observer into the tectonic space of structure.

The spatio-geometric development from isometry to axonometry allowed for the re-conception of parallel projection as a worm's-eye view. Whereas the oblique projections were intuitive, conventional, and planar, the invention of the axonometric introduced rational spatiality to the conception of the object. Axonometry, as opposed to isometry, encouraged Choisy's revolutionary view of tectonic assemblies, spawning the inside, up view. Ultimately, the worm's-eye view would prove to be the critical element which would allow Choisy to envision a novel graphic language by which to communicate the emerging theoretical notions of tectonic space.

The most uniform graphic device in the plates of both the *L'art de bâtir chez les Romains* and *L'art de bâtir chez les Byzantins* (1883) is the use of the worm's-eye view. In the *Romains* the up-view is applied to isometric, dimetric, and trimetric axonometrics and oblique projections from the plan and elevation, figures 5, 6. There appears to be no immediate system. The types and angles of projection are diverse. Perhaps following the recommendation by the Meyers, Choisy adjusted each viewing angle to that which best expressed the object to the observer. The oblique projectors of the drawings all vary as well.[17] The drawings, oblique and axonometric, are all considered projectively, each

established with an axis. Choisy is clearly experimenting with parallel projection. Notably, the plates in the *Byzantins* shows no such experimentation; representations are limited to canonical isometric and oblique projections. Regardless, the twenty-four plates of the *Romains* and the twenty-five plates of the

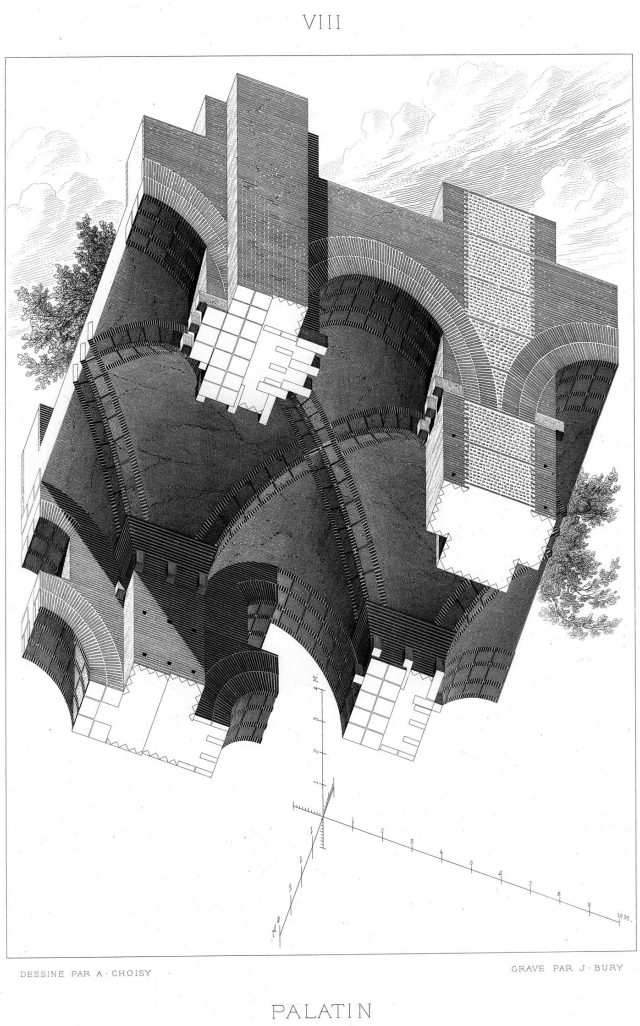

Figure 5
The plan is parallel to the projection plane in this military oblique projection (Choisy 1873, pl. 8)

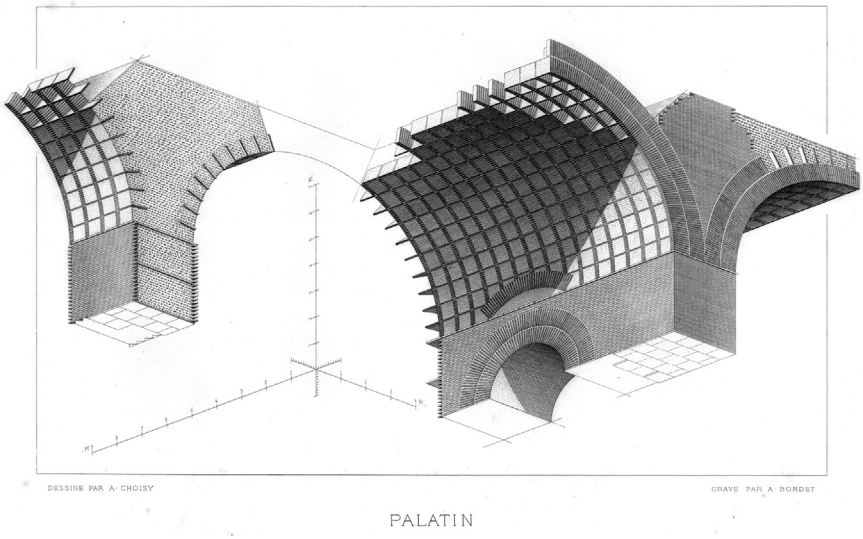

Figure 6
A dimetric axonometric projection (Choisy 1873, pl. 1)

Byzantins can be almost equally divided into two types of parallel projection: oblique and axonometric.

At this time, the two drawing types are minimally differentiated. It appears that the development of the axonometric allowed Choisy to similarly consider the oblique projection. The plates clearly articulate Choisy's equal experimentation with the dual arms of parallel perspective posited by Gournerie when Choisy was a student. The one rule which can be adduced at this time is that the tectonic space of the vault is best expressed from below.

A sketch of the up-view

After the 1883 publication of the *L'art de batir chez les Byzantins*, a great part of Choisy's research efforts were devoted to the development and production of the *Histoire de l'architecture*.

Choisy's sketches from 1885 reveal the nature of his evolution in his use of parallel projection from the *Byzantins* to the *Histoire*. His travels that year are represented by sketches of the churches of Saint-Etienne de Nevers, Abbaye de Saint Savin, and Notre Dame la Grande de Poitiers. Within the pages of the *Histoire*, all three buildings similarly represent a stage in the formal progression of

Romanesque architecture. But, there are noticeable differences among their sketched representations. The sketch from Nevers is raw, figure 7. Both the textual and the graphical notations are smudged by an error and reflect a correction. The line work of the sketch is loose, perhaps even inaccurate. This might be due to haste, uncertainty, or technique. The viewpoint is from below; the view conveyed is of the apparent scene, like that of the visitor standing in the aisle or perhaps sitting at a pew and looking up. The sketch shows an on-site impression of the structure. The other pair of sketchbook pages from 1885 contain a study of Saint-Savin on the left page and Poitiers on the right, facing page; both sketches approximate canonical isometrics, figures 8. Furthermore, they are both upviews, like the sketch from Nevers. Unlike Nevers, the view point is more analytical; it moves below the horizon line and the constructive elements are suspended in space. Under this aspect, these two sketches, St. Savin and Poitiers, are unique. There are no other extant sketches that so closely correspond to their future, final

Figure 7
St. Etienne de Nevers. Choisy, sketchbook, 1885 (Royer 1960)

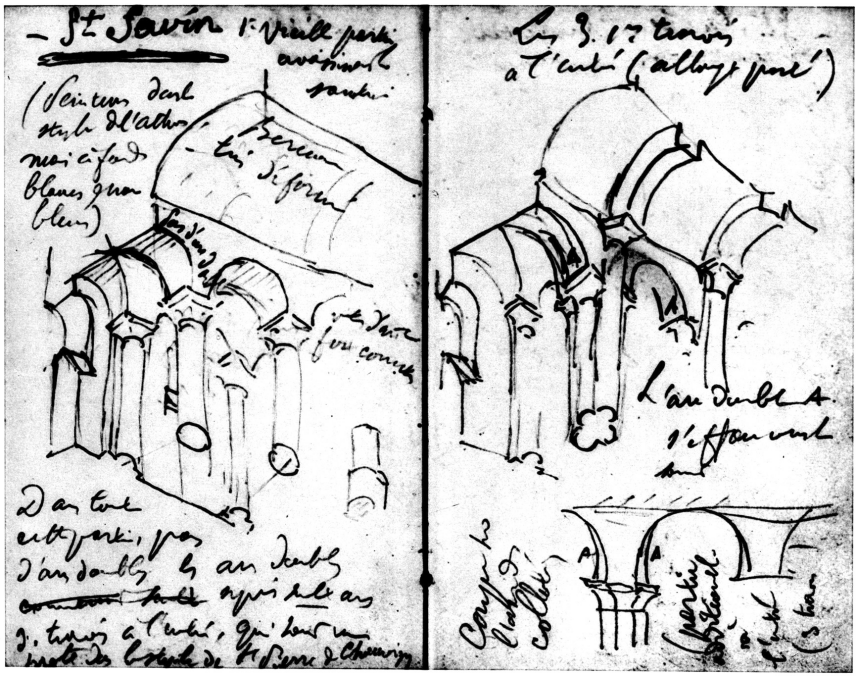

Figure 8
Eglise de Saint-Savin, *left*; Notre-Dame de Poitiers, *right.* Choisy, sketchbook, 1885 (Royer 1960)

composition as published in the *Histoire* fourteen years later, figure 9. One can easily speculate that these juxtaposed designs form a prototype and reflect Choisy's proposed, and ultimate, comparative format for the *Histoire*. Between the sketch and the illustration, the chronological adjacency, viewing angle, and level of detail remained virtually unchanged. Most notably, the form of parallel projection is changed, from isometric axonometric to oblique projection. One can reasonably draw the conclusion that Choisy's transition from axonometric to oblique projection in this case is fully intentional, and that each drawing system was used for particular ends in the *Histoire de l'architecture*.

Unlike Choisy's flexible and extensive variety using oblique and axonometric projection in the *Romains*, the *Histoire* shows limited, controlled and directed use of the two drawing types.

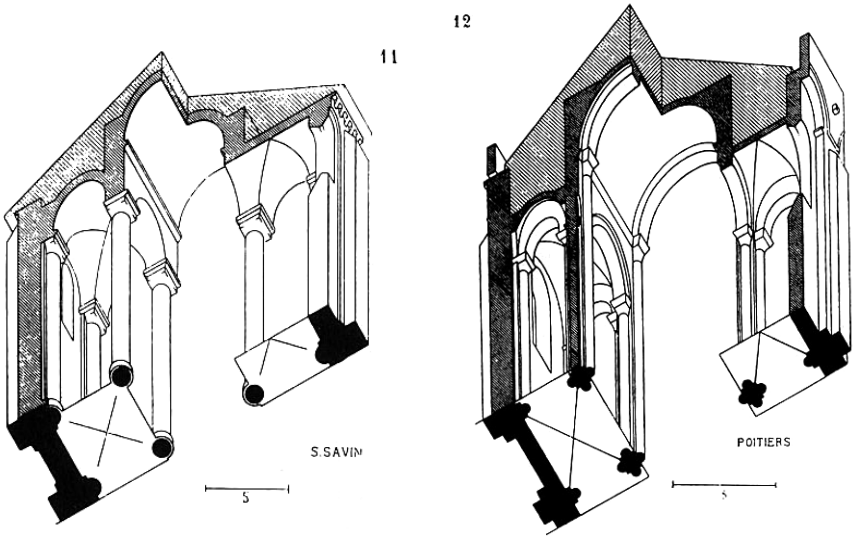

Figure 9
Eglise de Saint-Savin, *recto*, p. 203; Notre-Dame de Poitiers, *verso*, p. 204 (Choisy 1899)

Measuring the qualities of Choisy's oblique projections

Oblique projections are used extensively throughout the *Histoire de l'architecture* to examine small scale details and large scale arrangements, via bird's-eye and worm's-eye views, generated from both *de front* and *du plan*. The oblique projections convey the constructive aspects of the history of architecture. They isolate critical elements of assembly but also composite assemblies. Choisy's oblique drawings emphasize likeness, repetition, pattern, and order; they are used for standardizing purposes, both macrocosmically and microcosmically. Choisy exploits the oblique projection for its intrinsic comparative, standardized disposition.

Oblique projections are predicated on an originating geometrical plane of projection. While the oblique projections represent three measurable dimensions of an object, they are not an accurate drawing of the object in space. Oblique projections do not maintain the spatial accuracy of the object in space as an axonometric does. The relative angles between the three dimensions are conventional or arbitrary, and inaccurate even if the scales of measure are projectively determined. The oblique projection is not a representation devoted to spatial accuracy but rather gives priority to the accuracy of the orthogonal projection plane. Depth is an illlusion relative to the orthogonal, abstract figuration.

Similar to the stereotomical cavalier perspectives, constructive elements are drawn with oblique projection. One can see this in Choisy's drawing of the old portico of the grand temple of Selinonte and it is confirmed in his textual description of it; «the *antae* T is presented in the form of a pilaster with squared section» (Choisy 1899, I, 328).[18] Choisy's cavalier projections also rely on the precision of the orthogonal horizontal plane to depict large scale architectural structure, such as the Egyptian Temple at Thebes (Choisy 1899, I, 64). Dynamic spaces such as domes, previously seen via spatial, axonometric projections, can be found rendered via oblique projection. When the pure geometry of the plane of projection is the important determinant of the architectural form, that plane maintains its essential orthogonal attribute. This is seen in the Pantheon, where the geometric circularity of the drum is paramount, and in the Greek and Latin cross plans of the Renaissance churches. The planar oblique views are used to uphold geometric truths. This synthetic quality is what makes them particularly compelling as a graphic language, symbolically communicating the theory of architecture.

Choisy guided this previously undeveloped and practical aspect of the oblique projection, or «cavalier perspective», toward a discrete theoretical purpose. The undistorted orthographic plane, the plan in the case of the *Histoire*, is established as a datum, figure 10. The typifying monuments of each epoch are generalized by military oblique projections which collectively operate as a graphic language to conceptualize the constructive architectural ideals fundamental to every civilization. Thus, despite appearing at irregular intervals throughout the long text, these inverted representations of architectural form establish a powerful presence from their systematic use. The undistorted plan, the core comparative element, becomes a standard measure of formal changes across time.

The worm's-eye view military oblique projections in the *Romains* and *Byzantins* typically sprang from a horizontal section well above the ground plane, not from a plan. These up-view drawings detailed the technology of the fragment and were limited to explicating particular vaulted conditions built into space. The case of the particular was illustrated. It was not until the *Histoire* that the structural element was posited in direct relation to its composite structure which is determinant of the whole form. This shift was contingent on the worm's-eye view moving from a more pictorial up-view to a completely imaginary position below the horizontal ground plane. The systematic use of the worm's-eye plan turned architectural form inside-out, laying bare the essential elements and spaces which reflect each civilization's tectonic spatial character.

Conformity of method was used to further discriminate essential differences. The rendering techniques, abstractive in nature, aided the comparative reading of the buildings. The measured uniformity of the conventional plan was combined with uniformly applied rendering techniques that reinforced the comparative na-

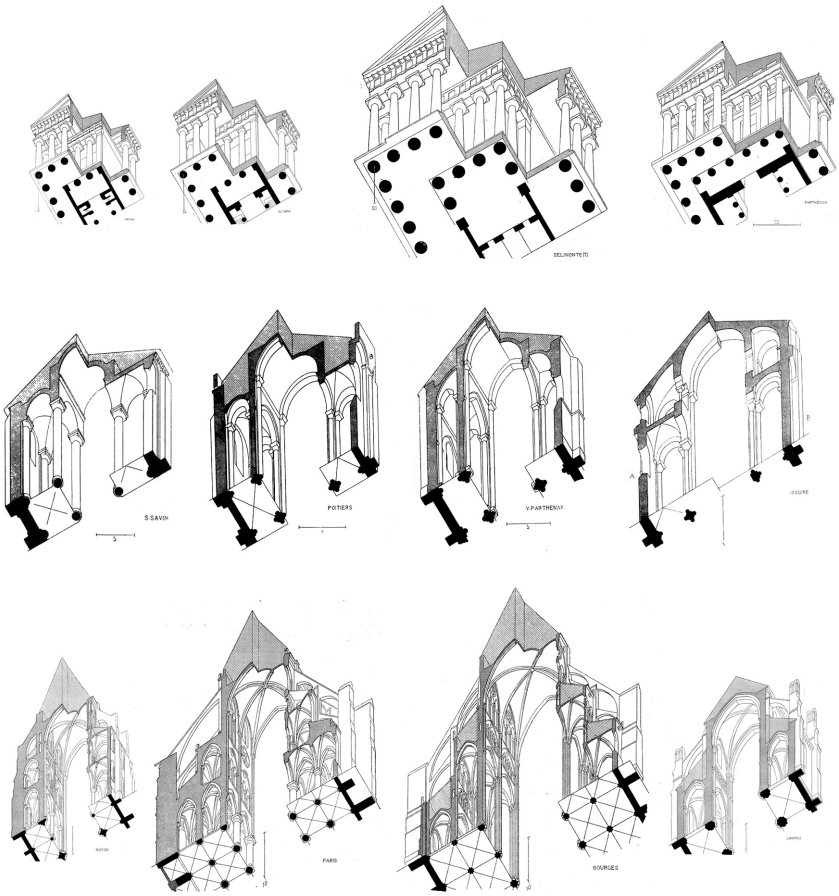

Figure 10
Worm's-eye view, plan based, oblique projections convey the spatio-tectonic progression of Greek, Romanesque, and Gothic architecture. Figure 10.1, Paestum, Olympie, Sélinonte, Parthénon (Choisy 1899, figs. 7, 8, 10, 11); figure 10.2, St. Savin, Poitiers, Parthenay, Issoire (Choisy 1899, 11, 12, 13, 16); figure 10.3, Noyon, Paris, Bourges, Langres (Choisy 1899, figs. 6, 7, 8, 10)

ture of the similarly oriented worm's-eye view oblique projections. The illustrative, individual documents of the *Romains* and *Byzantins* became necessarily abstracted to posit a conceptualized theory: rendered materiality was shifted to a structural framework, the temporal shadows were replaced by reasoned *poché*, the subjective orientation of three dimensional axes was supplanted by the re-

peated system of the oblique, the fragment of the ruin became a module of design. The new graphic methods transformed the artifact into a proposition. The history of architectural form was imaginatively contemplated through rational and scientific techniques.

Furthermore, Choisy refined his dissections of the artifacts of the past. His sections are controlled and specialized relative to revealing different aspects of tectonic organization systems. Dissections and cuts are performed on the same parts of the same building types in a deliberative process to not just reveal the bones of the interior, but to impart a particular reading of the spaces related to a particular skeleton. Oblique projections in the Greek, Romanesque, and Gothic divisions read like prepared microscopic slides. The depth is confined and defined by the module of the bay in the Romanesque and Gothic churches. All three reveal a spatial organization which is contingent upon their respective structural system which is reflective of the cultural system. The central nave soars with the support of the cross-vaulted structure flanking each side in the Romano-Goth churches; the eventual separation of wall and column corresponds with opening to the sky in demarking the sacred Greek cella. The sectioning technique was moved from the lateral to the vertical plane to reveal the different tectonic structure of the Greek façade, figure 10.1. The cuts reveal the layering of the columns as they wrap around the cella, in a progression of slices stepping back from the front portico. The plans also succumb to the incision of sectioning as they further establish fundamental hierarchies in form and structure. The Romanesque and Gothic church plans are variably cut through into the central nave communicating the primacy of this volume; the secondary volumes, the side aisles, are closed by cross-hair lines reflecting the module. The Greek cella drawings are similarly cut to reveal the actual spatial connection between earth and sky. The sectioning reveals the central tectonic character of the architectural space of each epoch.

For Choisy, dissection became an intellectual method of discursive thought, as a means of hypothesis. The deconstruction indicated construction. The purification of parts, elements, modules, proportions, hierarchies, bays, and systems, indicates a reverse process, one of building. Thus, Choisy defied the general trend of separating the real from the imaginative. Choisy uses the synthetic simultaneity of the oblique projection symbolically, to represent building, in addition to buildings.

Building his modern vision was achieved through these individual and sequential graphic constructs. Because the oblique projections were limited spatially due to their obligatory relation to the flat geometrical plane, the drawings could achieve a compelling, artificial spatial simultaneity. Each individual image, working from the inside out, simultaneously builds in three dimensions, montag-

ing the plan, the section, and the elevation. Linear progression is also a montage; the collective of individual drawings, similarly rendered, juxtaposes the variability and evolution of construction over time. The uniform oblique worm's-eye projections establish visually the possible combinatorial array of fundamental constructive ideas. Choisy made this evident at the conclusion of the linear demonstration of the structured forms connected through the bay of Gothic structure:

> At this moment all possible combinations are exhausted: the flying buttress gave daring solutions to direct lighting, the structure of buttressing with simple abutments furnished buildings with triple, double, and single naves; the general history of the bay should stop here (Choisy 1899, II, 468).

The montages of this architectural structure, seen individually and communally in Figure 10.3, symbolically manifest the act of construction. The worm's-eye view oblique projections are conceptual and multivalent. These representations transcend their measured objectivity to construct imaginatively a theoretical, didactic domain. The power to abstract a theory of architecture in a graphic form is that which makes it useful. The theory only thus transcends the particular and leaves the realm of the imitative. The essential rule is abstracted and thus proposed for the basis of future action. The *Histoire* does not document the history of architecture, demonstrate the history of tectonic form, or propose an historic ideal; rather the *Histoire* establishes the relentless but varied expression of constructive matters in all architectural form. A theory of making is revealed in the graphic history of architecture. Under each possible principle there is an array of many possible forms, each particular to its time and place. Choisy's history does not dictate, but provides a theory, based on generalizing principles, by which to approach the art of building.

Measuring the qualities of Choisy's axonometric projections

Whereas the innate attributes of Choisy's oblique projections were used to manifest comparison, sequence, construction, assembly, repetition, order, elements, disposition, and organization, the intrinsic spatiality of axonometry was applied to other ends. Choisy utilized isometric and dimetric projection to communicate the effect of architectural form *in* space and *as* spatial. The axonometric embodied architecture's movement, spatiality, animation, harmony, variety, character, and sculptural qualities. In one simple illustration, the constructive techniques used to place the massive stones of the Greek temples were animated or set in motion using axonometric representation (Choisy 1899, I, 273). The axonomet-

ric, while still exposing practical rationality, leaned on its oblique attributes to render impressions or views. Finally, the axonometric was reserved to represent the ideal spatial expression of tectonic form, as in his representation of Sainte-Sophie de Constantinople.

As established by its history, the axonometric is a contingent condition of free axial rotation of a body in space; the body's representation, however, is inclined relative to the projection plane. Ideally, Choisy could have posited the objects of the *Histoire* at any angle of rotation, but the greater part of Choisy's axonometric representations are isometric and dimetric. The isometric in particular is the most easily rendered axon, due to its axial equilibrium, but it also heightens spatial depth. Within the *Histoire*, the isometric is privileged over other inclined views which inevitably favor one of the axial planes. The worm's-eye view isometric equalizes the three dimensions of the object with the space of the object. Within orthographic projections the relative angles of the object in space are true, but the projection's perpendicular relationship to the plane of projection skews the view. The plan, kept orthogonally pure in the oblique (military) projections, is skewed but no less accurate and measurable in the axonometric projections. The axonometric's explicit warped view is intentionally used by Choisy to animate his graphic visualizations. At times this agitation is used to reflect a spirit of dynamism, while at other times it indicates a sense of disturbance.

Choisy's measured use of axonometry to convey qualities of perception seems to extend naturally from his considerations of point of view first made evident in his study of the Acropolis. Choisy used perspectival representation to illustrate and parallel the nature of visual perception that guided composition by the Greeks. Similarly, the worm's-eye position emerged from his considered perception of an observer within the rational space of construction. Choisy considered human perception as vital to the art of architectural structure and he subtly manipulates our perceptions through his reasoned graphic constructs. In the *Histoire*, Choisy tuned his use of the axonometric to impart perceptions of the character of built form, particularly as a spatial construct. Choisy did not advocate that character was a primary determinate of architectural form, rather he posited that architectural tectonics transmit sensations or effects that are perceived. A worm's-eye view axonometric model allowed these traits to be perceived similarly. Additionally, Choisy contrived axonometric representation to transmit his interpretive understanding of the effects of architectural form; his axonometric representations are not objective documents of historic buildings but subjective, imaginative models of an integrated vision of the spirit of structures in space.

The poetic physiognomy and expressive character of Greek decoration was communicated by Choisy with axonometric views, figure 11. Choisy graphically

details the theory on the Greek's transition from wood construction to its mimesis in stone. Isometric worm's-eye view drawings represent the perfection of such effects and their expressive character found in columns and entablatures. The three dimensional spatial perfection of the elements rising in space communicated these effects—not atmospheric renderings or perspectives. The intentionality of Choisy's mode of representation is clear; most columns in the *Histoire* are represented by oblique projections to precise constructive and geometric aspects of the elements. Thus, the unique use of isometry to disclose the decorative refinements of Greek architecture is clearly intentioned. Through the use of axonometry, Choisy relates the poetic, emotive aspects tied to Greek architecture. The art of building was not just the result of science but the manifest expression of culture as well. Choisy used the axonometric to stitch together the cultural and the constructive.

The bird's-eye view isometric representation of the mass plan of the Brahman temples exemplifies Choisy's intentioned use of the axonometric to render a spirit of movement and variety. Choisy explains that temple complexes were constituted by successive additions of temples and enclosures in a concentric growth from a single temple over time (Choisy 1899, I, 174–5). The cited source material, *Essay on the architecture of the Hindus* by Ram Raz, depicts a complete plan in a geometrical scaled drawing, figure 12. Evidently, Choisy did not find the essence of the place reflected via a measured survey, by a rigid symmetry, or a summary view of clustered masses; he was concerned with imparting the «effects of the pylons» (Choisy 1899, I, 175). The factual conditions were essential-

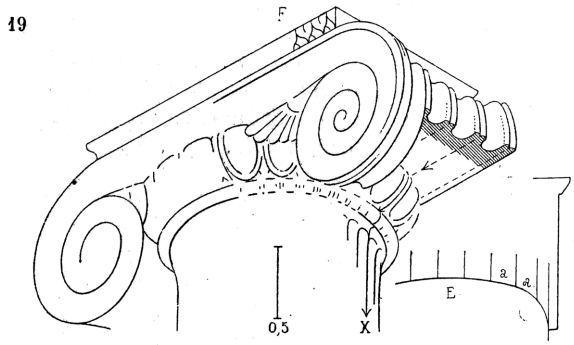

Figure 11
The delicate physionomy of the ancient Ionic capital at Ephesis represented the perfection of traditional form and the expressive character of the Greek art (Choisy 1899, 1: 356)

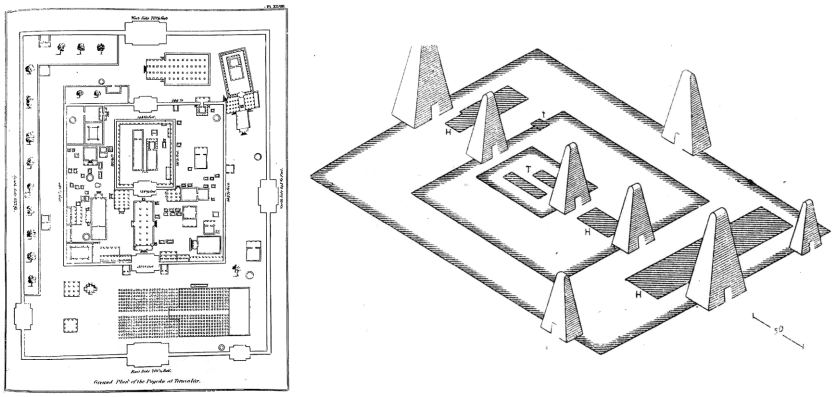

Figure 12
Figure 12.1, plan of Pagoda at Tiruvalur (Raz 1834, pl. 48); figure 12.2, Hindu Temples (Choisy 1899, 1: 175)

ized and walls, trees, decoration, support structures eliminated or abstracted. Choisy described a «sentiment of crushing majesty» imparted by the temples' monumentality and their profuse ornamentation (Choisy 1899, I, 175). This sensation is rendered via the use of the axonometric to perturb the plan; the monumental pylons anchor and order their space.

Choisy's axonometric representation of the effects of the pylons relative to their plan and absent their ornamentation was singled out twenty years later by Le Corbusier. He included Choisy's representation of the Hindu temples in *Vers une architecture* with the caption, «The towers create a rhythm in space» (Le Corbusier [1923] 1995, 35; Goodman translation, 117). Corbusier observed,

> Rhythm is a state of equilibrium arising from simple or complex symmetries or from skillful compensations . . . So many fundamentally different reactions on the individual, despite the unity of aim that is the rhythm, that is a state of equilibrium. Hence the astonishing diversity of the great periods, a diversity that comes from their architectural principle and not from their ornamental modalities. (Le Corbusier [1923] 1997, 37–8; Goodman translation, 119–20)

Choisy concluded that the formation and effects of the Indian temples were parallel to those of Egypt and China, each of which were similarly represented via isometric drawings (Choisy 1899, I: Karnak, 59–60; China, 192). The seem-

ingly simple sequences and rigid, ordered symmetry within the temple enclosures at Karnak were declared to be imaginatively and intentionally programmed toward a «gradation of effects and an impression of mystery» (Choisy 1899, I, 60). These effects were a measured darkness of the central path which increases as the ground rises and the ceilings lower on approach to the sacred sanctuary. The mystery within the rigid plan was indicated by an axonometric view, not a mimetic or atmospheric perspective. Choisy's impressions of the variety within the intentioned patterns of architectural form ordered in space and over time are communicated via the biased view, the axonometric. The warping of their respective plans turns the focus away from perceiving their structure as merely the result of a geometric pattern.

Choisy's emphasis of the perception of the effects of form in space leads to the hypothesis that his worm's-eye view axonometry takes the place of linear perspective; his axonometry is the constructive perspective. Choisy offers a dynamic vision of constructed space for the eye of the rational observer. His axonometrics bivalently model the effect of architectural form in space and as spatial. Like the building itself, axonometric drawings are constructed and scaled three-dimensionally, thus Choisy's representations are indeed lively and animated like the three-dimensions of architecture itself.

Choisy did not explicitly speak of architectural «espace» in the manner understood today following its theoretical explication during the twentieth century, but he graphically established the use of the axonometric relative to such notions of space. The aesthetics of the absolute concept of three dimensional space were made explicit in the *Histoire* through Choisy's worm's-eye view axonometrics. These are seen in greatest number in the representations of domed spaces. The activated «space» of axonometry is not applied uniformly to all domes or vaults; in fact, the domed spaces of the Renaissance are rendered via oblique projections. Choisy explicates that the domes created by the Renaissance architects are limited by, and subordinate to, the plan. Thus, the orthogonal nature of the plan's geometry is maintained. Choisy makes clear that domed construction is perfected in buildings in which the plan, the order, the elements, the structure, the proportions, and the materials harmonically rise into space like an embodied organism—these are the drawings which necessitated rendering a full impression. The axonometric is used to animate such edifices in space.

Choisy uses an isometric projection to evidence the harmonic totality of the unified structure in the domed space of Sainte-Sophie de Salonique, figure 13. He considers that the whole composition grouped itself around the dome, writing: «The whole system of equilibrium is inside; all the dispositions are considered in light of supporting and accompanying the dome: the ensemble, in which each part is subordinate to the principal reason, produces an impression of strik-

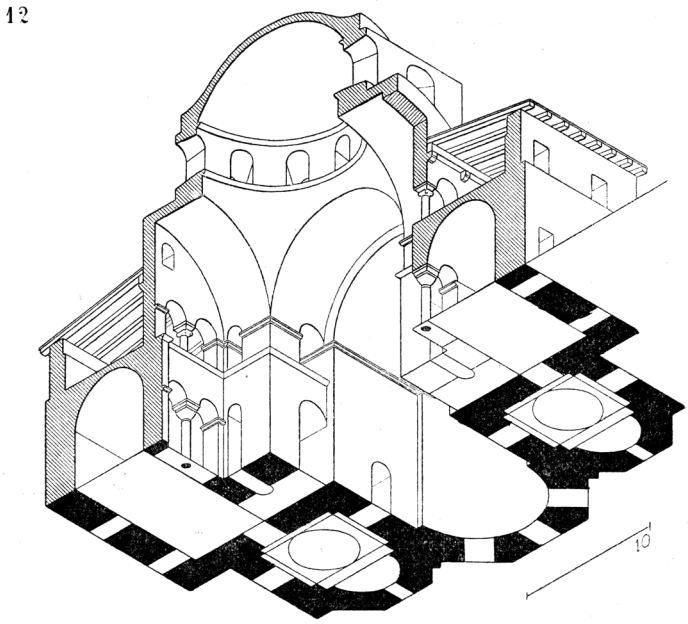

Figure 13
Sainte-Sophie de Salonique, bird's-eye view isometric (Choisy 1883, 2: 23)

ing clarity» (Choisy 1899, II, 47–8). The constructive ensemble and the space of the dome are impressed upon the reader with the spatial simultaneity of the worm's-eye view axonometric. No other drawing type could have reproduced this effect. Choisy used the isometric, the most harmonic and balanced axis system, to express the diagonal presence of the thrust that the pendentives transmit to the corner mass (Choisy 1899, II, 48). This drawing was not the first that Choisy had made of Salonique. He had included the building's representation in the *L'art de bâtir chez les Byzantins* eighteen years earlier, figure 14. Both representations are isometric, however the worm's-eye view in the *Histoire* more clearly manifests not only the constructive principles, but also the space of the structure. The former, bird's-eye view, rendered a much clearer conception of the whole building. The complete plan was indicated and one quarter of the building mass was depicted. The pendentive in the corner was the focus. However, the volume of the enclosing structure is nonexistent, distilled to an ellipse in the center of the plan. The latter, worm's-eye view, soars into space, evoking the spirit of the building's essence the space of the structure.

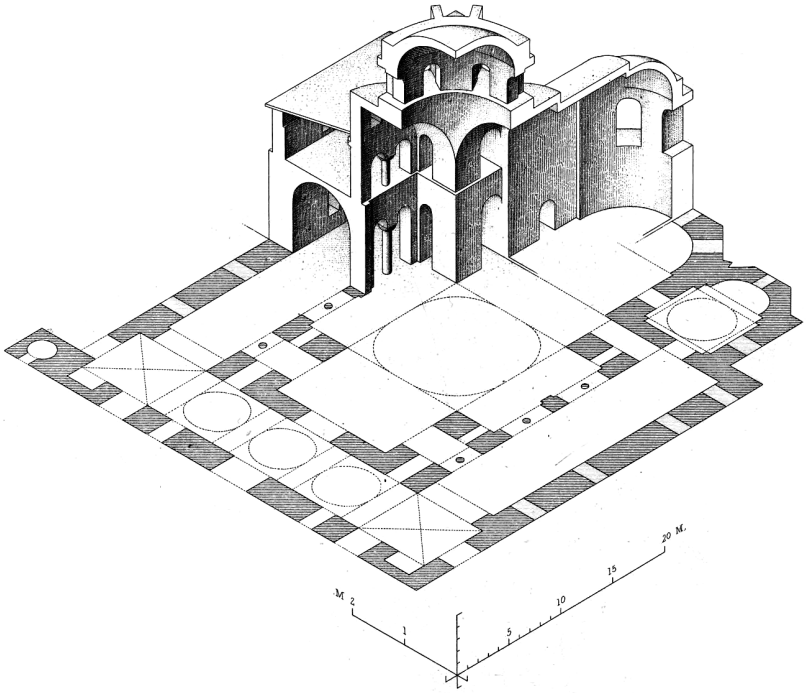

Figure 14
Sainte-Sophie de Salonique, worm's-eye view isometric (Choisy 1899, 2: 48)

Sainte-Sophie de Constantinople is upheld by Choisy as the pinnacle of domed Byzantine construction, exhibiting the ultimate in architectural unity, figure 15. Again, Choisy focuses on the effect of total composition, citing the building's harmonic effects of the structural «mis-en-scène» (Choisy 1899, II, 50). The audacious Sainte-Sophie embraced both the art and the science of architecture. The ideal of the circle and the square were spatially, constructively, and decoratively harmonized. Choisy describes the poetry of the structure of the central dome: it was isolated by illumination, «as suspended in space» (Choisy 1899, II, 51). His axonometric projection shows a free, floating image, detached from the ground plane. The building in space and the space of the building are animated.

The skewed view attaches a bivalent space to the structure of architecture. The axonometric imparts a sense of the thing. The tectonic object is qualified as an impression. These axonometrics are not objective, practical images; they transcend their technique through their techne to reveal a conditional truth. The ax-

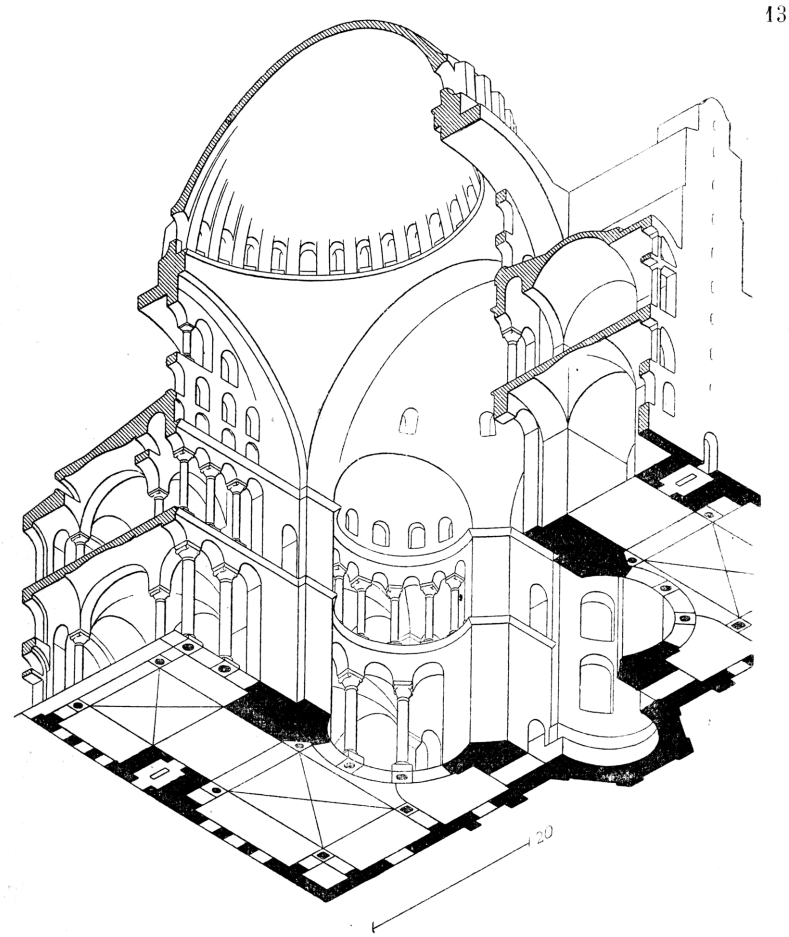

Figure 15
Sainte-Sophie de Constantinople, the Byzantine monument *par excellence* (Choisy 1899, 2: 49)

onometric, unlike the oblique projection, is a model. It acts like the thing itself. The drawing's spatial purity is imparted to the structure itself. Space becomes a necessary architectonic element.

The axonometric's spirit of construction, in the drawing as in the artifact, reveals beauty in the measured truth of technique. The analyses of the critics variously reproach the aesthetic problem of essence versus appearance. Yet, the

worm's-eye viewer engages the infinite, homogeneous, axonometric space embodying the building's objective reality and the effect is one of qualitative specificity. The space of the structure is animated by the observer's rational point of view. The worm's-eye view axonometric image does not parallel man's visual perception but approximates it. The drawing models the spatial reality of the object and man's synthetic and moving perception of it. The eye does not rest fixedly even when the man does. The eye hovers, repositions, and realigns, creating visual mental montages. The axonometric offers a graphic vehicle that reflects the modern position that an observer's perception is synchronic and multivalent. The nature of axonometric space allows for the graphic representation of architecture that approximates this dynamic reality. The apparent image of something may or may not reflect the essence of the thing. As Scolari reminded us, Plotinus allowed that the image should be a reflection of the thing, sharing the same nature as its mode (Scolari 1985, 77). Choisy's axonometric images follow this type of imaging and imagining.

The measure of qualities

Unlike Choisy's drawings in the *Romains* and *Byzantins*, the oblique and axonometric drawings in the *Histoire* are not projections. This is subtly indicated by the disappearance of the axial, spatial scale for each dimension and its replacement with a single dimensional scale for all three. Both the oblique and the isometric drawings are 1:1:1. In this way they are both iso-metric, or of equal measure. However, Choisy clearly directs the phenomenal differences of each measure to different qualitative ends.

Notes

1. The first part has been dated to 1852, a second, 1853; and a third, 1855 by Robert Schmidt, among others. Martin Herrmann Meyer died in 1856. His brother C.Th. Meyer continued on with a fourth section, publishing all sections together much later under the title*: Lehrbuch der axonometrischen Projectionslehre (1855–1863).* C.Th. Meyer is critical of the intervening works by the civil engineers and Robert Schmidt, portraying them as plagiarists. This criticism seems to be warranted against Schmidt's work.
2. «Eine Darstellung, welche zugleich einen tieferen Blick in die Einrichtung, relativen Verhältnisse und Masse gestattet».
3. «Je nachdem man dabei dem Auge einen gewissen Standpunkt giebt, oder bloss gewisse angemessene Regeln anwendet, unterscheidet man perspectivische und freie oder fingirte Projectionen».
4. «Malerperspective».
5. «Bei welcher das Auge über jedem einzelnen Punkte stehend».

6. «Dass man auch bei diesen Axensystemen eine Neigung nach hinten vornehmen kann, so dass der Beschauer gleichsam schräg von unten in das System sieht, bedarf seiner weiteren Erwähnung».
7. Plate 3; Figure 37 a, b (1:1:1/2); 37 c, d (1: 1/2:1); 37 e, f (1/2:1:1).
8. The oblique drawing and isometric projections were from Haindl's *Maschinenkunde und Maschinenzeichnen*, 1843.
9. Meyer and Meyer acknowledged the special case of the oblique projection which has projectors making an angle of 45 degrees planar within the projection plane. They concede that the geometry of the axes seems to coincide with the geometry of an isometric; it is nevertheless an oblique projection. This cavalier condition does not have equal measure along the three axes. They assert that one should use caution not to confuse the two.
10. «Das durch dieselbe entstehende Bild ist gleichsam der orthographische Schatten». This contention is repeated throughout the volume.
11. Meyer and Meyer's publication was in the library of the Ecole Nationale des Ponts et Chaussées, see: *Catalogue des livres composant la bibliothèque de l'Ecole des Ponts et Chaussées*, 1872.
12. The translation of the text appeared in four installments in 1843: January, columns 3–14; July, columns 289–304; November, columns 481–507; and December, columns 529–537. The plates appeared in random order: 2 in July; 12, 17, 19 in November; and 16, 18, 20 n.d.
13. An earlier publication by Willis, *Remarks on the Architecture of the Middle Ages* (1835), contains one plate containing one trimetric and two isometric drawings which are also likely to have influenced Choisy.
14. «Cette ossature empâtée dans le corps des maçonneries».
15. The angles of the dimetric are 130, 130, 100.
16. The angle is very close to 45 degrees from horizontal.
17. 22/68, 68/22, 15/65, 45/45, 35/55.
18. «L'ante T se présente comme un pilastre a section carrée».

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